

## CEN: Thomas Ströhlein’s Endgame Tables, a 50<sup>th</sup> Anniversary

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We should not let February 2020 recede too far into the distance without celebrating the 50<sup>th</sup> anniversary of Thomas Ströhlein’s (1970) Ph.D. thesis, *Research on Combinatorial Games*, see Fig. 1. Previously, Bellman (1965) had indicated that Dynamic Programming could be applied to endgames. Ingo Althöfer (2019) relates that the topic was proposed by F.L. Bauer after the backwards analysis of games and puzzles had been mentioned to him in the Netherlands by two Dutch colleagues, Max Euwe and Wim van der Poel (van den Herik, 2020).

The thesis considered perfect-information, win-loss games using the concepts and results of graph theory and boolean matrices. The properties of winning and optimal strategies were then described. After defining Graph Kernels, Ströhlein brought chess into scope and described the first realisation of a retrograde algorithm to create endgame tables. In the last of nine chapters, results including correct maximal depth figures<sup>1</sup> were presented for the five pawnless endgames KRk, KQk, KRkb, KRkn and KQkr. While Bellman’s Dynamic Programming (1965) noted that optimal endgame play could be defined, Ströhlein’s computational graph-theory discovered, illustrated and analysed it.<sup>2</sup>

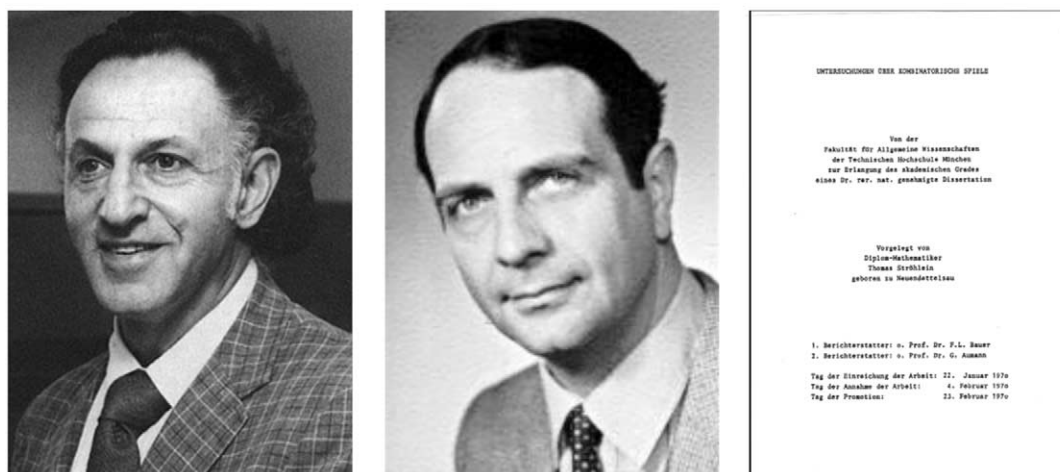


Fig. 1. Richard Bellman, Thomas Ströhlein (CPW, 2020b) and the title page of *Research on Combinatorial Games*.

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<sup>1</sup>TS’ figures in today’s depth-to-conversion notation ‘DTC’: KRk (maxDTC = 16 winner’s moves), KQk (10 moves), KRkb (18m), KRkn (27m) and KQkr (31m). Computers can quickly prove that Kk, KBk and KNk feature no wins.

<sup>2</sup>Bellman (1965) notes that two positions may be regarded as equivalent,  $P_1 \sim P_2$  in the sense that one can move from either to the other. However, the equivalence classes so defined do not quite correspond to all the positions of an endgame force. There is, for example, no KNNk position that precedes or succeeds 8/8/8/8/1NN5/2K5/k7 b.



Fig. 2. 1964: the LRZ, Richard-Wagner-Strasse 18, and the AEG TR4 (Bauer, 2007).

The actual computations were carried out in the period 1967-9 (Schmidt and Ströhlein, 1989, 1993). These were the first years of a West German National Research Programme. They were also the last years of the AEG-Telefunken TR4 computer at the 'LRZ' Leibniz Rechenzentrum, see Fig. 2 (Bauer, 2007; Bitsavers, 2007; CPW, 2020a/b; Sapper, 2020). Lest we forget, this computer's 0.25MB of core memory, 50MB of disc, and speeds of 4.5/30 $\mu$ s for fixed-point add/multiply represented leading edge performance in Europe when it was installed for \$2.5m in 1964.<sup>3</sup>

Table 1

The thesis' table on p62 – plus column six

#	Endgame	md = maxDTC	~ comp. time t	t/md secs.	p > no. pos.	t/(md*p) msec.
1	KRk	16	9m	33.8	65,536	516
2	KQk	10	6.5m	39	65,536	595
3	KRkb	18	6h 30m	1,296	4,194,304	309
4	KRkn	27	14h 16m	1,902	4,194,304	454
5	KQkr	31	29h 9m	3,384	4,194,304	808

Ströhlein's computer model of chess simplified the code for practical reasons. The king was not mated but captured after being surrounded like Richard III. This capture could also notionally be done by the opposing king but that would have been captured first! 'Capture depths' on some print-outs were therefore one more than *dtc* depths in today's 'DTC' Depth to Conversion metric. With no pawns and with castling considered unavailable as now, the squares a1, a8, h8 and h1 were rightly considered equivalent. However, for simplicity of programming and of reading the output, the sides a1-a8 and a1-h1 were not, so Ströhlein's raw count of maxDTC positions is only slightly less than double the number of distinct positions. White is the stronger side and the focus is on wins for White, mainly White to move. The table on p62 of the thesis gives the correct maxDTC figures as in Table 1 here. Clearly, the step up from 3-man to 4-man endgames was a major one and a considerable feat worth pondering.

In addition to consulting the relevant KQk and KRk positions, the number of positions involved exceeded the number of bits in memory so data had to be managed to and from disc. Thomas credits

<sup>3</sup>Similarly, Cambridge University's ~\$5m 1965 TITAN ATLAS by 1968 sported 0.75MB core-store, 40MB disc and fixed-point add/multiply times of 1.6/5.0 $\mu$ s. \$2.5m in 1964  $\approx$  \$21m in 2020, 'top ten' petaflop money.

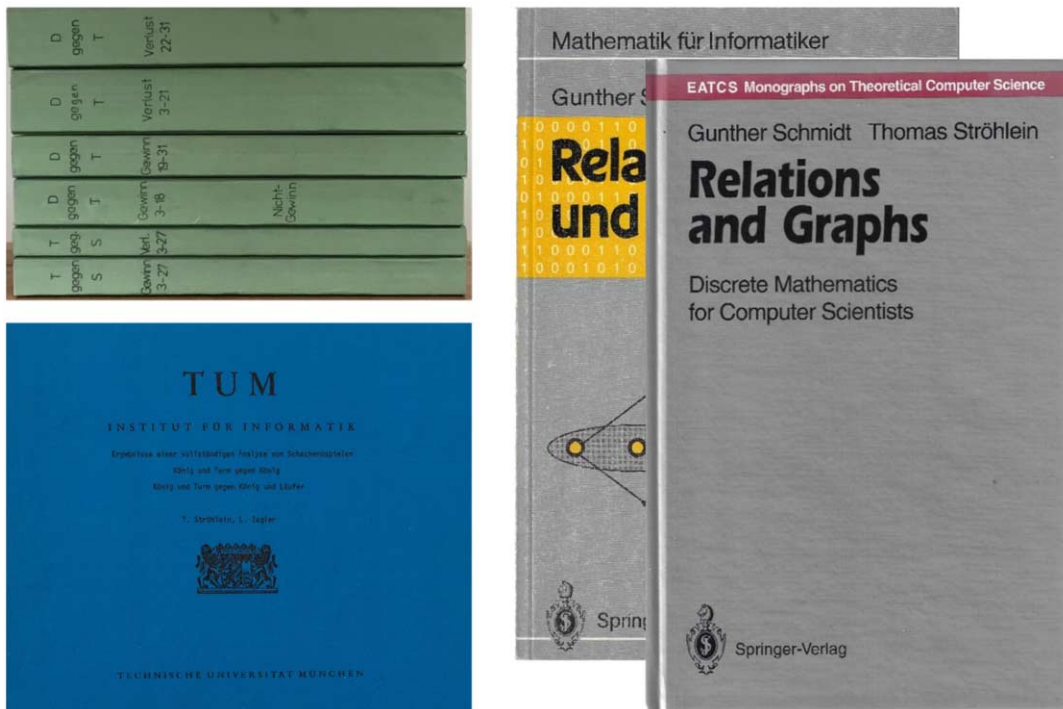


Fig. 3. The KRkn and KQkr results, 'TUM-INFO' (1978) and *Relations and Graphs* (1989, 1993).

his computer scientist wife, Ingeborg, with the finer points of the computing including the fast bit manipulation in machine code. Some remarkably long computer runs were involved: the TR4 was notably more reliable than its successor, the TR440 (Bauer, 2007, p. 102). KQkr later became the icon of non-trivial endgames thanks in part to Thompson (Kopec, 1990) and Jansen (1992).

Consideration of the computer results continued after 1970 in association with Gunther Schmidt, acknowledged in the thesis. The outputs for KRk and KRkb were photocopied and bound, see Fig. 3, and further analysed in Ströhlein and Zagler (1978) which included, see Figs. 4 and 5:

- 1) pp 003-088: all KRk positions with an optimal move; '!' indicates uniqueness,
- 2) pp 089-100: a list of KRk positions with  $dtc = 4$  and a unique optimal winning line,
- 3) pp 101: a list of the maxDTC positions, i.e., with  $dtc = 16$  moves,<sup>4</sup>
- 4) pp 105-202: a lexicographic list of all winning KRkb positions with  $dtc = 4$ .

A winning move is given: '\*' = 'only winning move' and '!' = 'uniquely optimal'.

Other work on relations, graphs and games (Schmidt, G. and Ströhlein, T., 1985; Ströhlein, T., 1976; Ströhlein, T. and Zagler, L., 1977) contributed to their definitive books on the subject (Schmidt and Ströhlein, 1989, 1993). These include the broader applications of games, one of which – program verification – is also relevant in the world of models and games.

Thomas Ströhlein's thesis and subsequent work has been an inspiration to later workers. We have since enjoyed Ken Thompson's sub-6-man 'EGT' endgame tables on CD (Tamplin, J.T. and Haworth, G.M., 2001) and the Nalimov (2000) 6-man EGTs online (Bleicher, 2010). We now benefit from sub-

<sup>4</sup>229 positions with  $dtc = 31$  ply: 121 distinct, being 108 pairs 'mirrored' in a1-h8 plus 13 exclusively on a1-h8.





Fig. 6. From left to right: Thomas Ströhlein, his wife Ingeborg and longtime friend and colleague Gunther Schmidt at the family '50<sup>th</sup>' celebration on 23<sup>rd</sup> February, 2020 of his 1970 doctorate.

note. I welcome any offers of help for my halting and inadequate attempts to do justice to his thesis in translation.

The e-version of this note (Haworth, 2020) provides supporting files including an archive on the TR4, the thesis, some extracts from references cited in the thesis, various pgn and data files, and the 40<sup>th</sup> anniversary celebration of LRZ in 2007.

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