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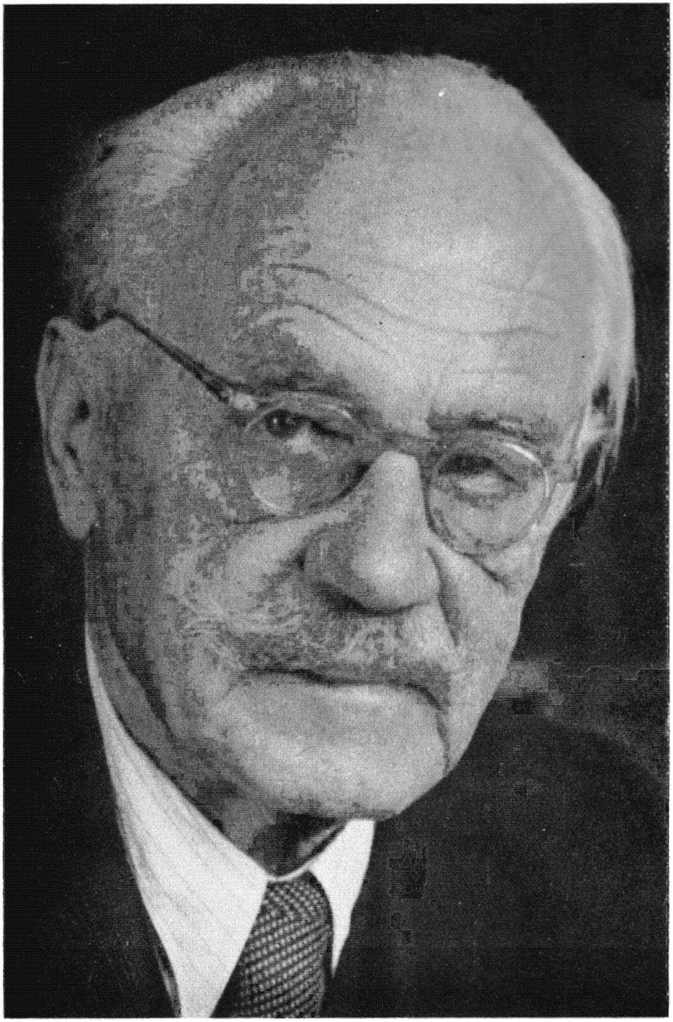

A. Sommerfeld

[Reprinted without change of pagination from *Obituary Notices of Fellows of The Royal Society*, Volume 8, November 1952]

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ARNOLD SOMMERFELD was one of the most distinguished representatives of the transition period between classical and modern theoretical physics. The work of his youth was still firmly anchored in the conceptions of the nineteenth century; but when in the first decennium of the century the flood of new discoveries, experimental and theoretical, broke the dams of tradition, he became a leader of the new movement, and in combining the two ways of thinking he exerted a powerful influence on the younger generation. This combination of a classical mind, to whom clarity of conception and mathematical rigour are essential, with the adventurous spirit of a pioneer, are the roots of his scientific success, while his exceptional gift of communicating his ideas by spoken and written word made him a great teacher.

I was not a pupil of Sommerfeld, but met him for the first time in later life. Yet from this moment our friendship was firmly established. The picture of his personality which I carry with me and shall try to describe in these pages is drawn on this background of personal contact. I have further at my disposal a charming autobiographic sketch, which he wrote in 1919 for the Academy of Vienna, with an appendix probably added in 1950, and several obituary articles, of which those written by Heisenberg and von Laue are the most remarkable.[1]

Arnold Johannes Wilhelm Sommerfeld was born in Königsberg, East Prussia, on 5 December 1868. His father, Dr Franz Sommerfeld, was a medical practitioner devoted to science and a passionate collector of natural objects like minerals, amber, shells, beetles, etc. Königsberg—now a Russian town with another name—was the capital and coronation city of the Prussian kings and had developed a specific atmosphere of erudition and culture. It was Immanuel Kant's place of birth and permanent home. The university of Königsberg was one of the first where theoretical physics became an acknowledged branch of study. This was due to the great authority exerted by Franz Neumann, who had founded an Institute of Theoretical Physics independent of and equal in rank with the experimental department. It is astonishing how many of the distinguished mathematicians and scientists of Germany's best period have come from Königsberg. When Sommerfeld became a pupil of the Altstädtische Gymnasium (High School) there were simultaneously attending Hermann Minkowski, Max Wien and Willy Wien. Sommerfeld says that at school he

[1] I have to thank Mr E. Sommerfeld, son of the late Professor A. Sommerfeld, for supplying a list of publications and the photograph reproduced here.

I am deeply indebted to Dr E. Ruch for helping me to compile the material for this article and correcting the text.

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was 'almost more' interested in literature and history than in the exact sciences, and that he was equally good in all subjects including the classical languages. In 1886 he passed the final examination and entered the university of his home city.

After some irresolution he decided to study mathematics, but also attended lectures on political economy and philosophy. The teaching staff in the mathematics department was at that time unusually brilliant; Lindemann, who later solved the ancient problem of the quadrature of the circle (by proving π to be a transcendental number) was the head, Hurwitz was assistant professor (Extraordinarius) and Hilbert, lecturer (Privatdozent). It was the inspiration radiating from these men which kept Sommerfeld from following the custom of German students to change their university. When he attended Hilbert's lecture on the theory of ideal numbers he came to believe that his interest was mainly directed towards the most abstract mathematics. Later he regretted having remained in Königsberg, as he was induced to join one of the students' societies (Burschenschaften) which absorbed a considerable part of his time.

Franz Neumann's successor in the chair of theoretical physics, R. Volkmann, seems not to have influenced young Sommerfeld. The centre of interest was at that time the transition from the older electromagnetic theories, based on action at a distance, to Maxwell's field theory, whose predictions had just been splendidly confirmed by Hertz's experiments. Sommerfeld found the inspiration, which Volkmann could not give, in a young scholar, seven years his senior, Emil Wiechert, who later became, like Sommerfeld himself, a master in electrodynamic theory, and well known through his work on the retarded potentials and through his pioneer work in seismology. In 1890 these two together devised a harmonic analyzer, which was built in Volkmann's institute (2).[2] It was of similar construction to that invented by Sir W. Thomson of which they, however, had not heard. The instrument was intended to be used for analyzing a set of temperature measurements at various depths at a station, founded by Neumann in the Botanic Garden. A prize was offered by a local scientific society for the best evaluation of these measurements. Thus Sommerfeld was led to the problem of conduction of heat (4). The case was difficult because the station was on the foot of a small hill. Approximating the surface by two intersecting planes, Sommerfeld reduced the problem to the solution of a linear differential equation on a Riemannian surface of several sheets, a method which he soon applied with great success to problems of optical diffraction. He competed for the prize, but had to withdraw his paper because of an error in the boundary conditions. In his recollections he remarks that this failure had its root in the characteristic attitude of this period to 'stick to mathematical generalities', instead of studying the peculiarities of the problem and using numerical methods.

In 1891 Sommerfeld obtained the doctor's degree in Königsberg with a thesis, 'The arbitrary functions in mathematical physics' (1), which he conceived and wrote down in a few weeks. With this paper he entered a field of mathematical

[2] Numbers in parentheses refer to the serial numbers in the bibliography.

study to which he remained faithful all his life, the representation of arbitrary functions by given sets of functions, for instance the eigenfunctions of partial differential equations. One volume of his Lectures, which he published during the last years of his life, deals with this branch of mathematics.

The first attempt 'to sail the high seas of theoretical physics proper', as he expresses it, was a paper entitled 'Mechanical representation of electromagnetic phenomena in bodies at rest' (3), which was stimulated by his reading of William Thomson's works. He modified Thomson's gyroscopic model of the ether by exchanging the part played by the electric and magnetic force. However, he soon became convinced that not much was gained by such mechanical 'explanations' of Maxwell's equations. Still the paper gave him the satisfaction of attracting the attention of Boltzmann. Sommerfeld's later attitude to the foundations of electromagnetic theory is clearly formulated in the first lines of the preface to volume III of his published Lectures, where he says that from his youth Heinrich Hertz's axiomatic construction of the fundamental equations had appeared to him as the model presentation.

In 1892 Sommerfeld passed the examination for the teacher's diploma and did then his year's military service. In October 1893 he went to Göttingen, the centre of mathematics in Germany. Through the accident of personal connexions he became assistant at the Mineralogical Institute under Th. Liebisch. But his real interest remained directed towards mathematics and mathematical physics. 'Overwhelming was the impression which I received, in lectures and discussions, from Felix Klein's grand personality.' Thus he describes the influence of that man whom he regarded as his real teacher, not only in pure mathematics, but also in his attitude to mechanics and mathematical physics. Well can I understand this feeling of devotion and awe which Klein inspired, as I came under his influence about eleven years later, at a time when Klein's magical powers as a teacher were perhaps at their summit. In fact, I found him too olympian, his lectures too perfect, his seminar discussions too encyclopaedic, and I preferred the more human personalities and formally less accomplished but livelier teaching of Hilbert and Minkowski. But these were not yet in Göttingen in 1893 when Sommerfeld appeared there, hence it is no wonder that he fell completely under Klein's spell. Many a time later we have discussed the merits of these three giants of mathematics, but Sommerfeld never wavered in his preference for 'the great Felix'.

In 1894 Sommerfeld became Klein's assistant for the management of the Mathematical Reading Room and its famous library. One of his duties was to work out Klein's lectures and to produce a copy for the use of students in the reading room. Thus he gained an intimate knowledge of Klein's characteristic method of lecturing which was of decisive influence on his own future teaching. Klein directed his attention towards the problems of mathematical physics and tried to transfer to him that attitude to these problems which he had laid down in previous lectures. The first fruit of this influence was Sommerfeld's thesis of 1896, 'Mathematical theory of diffraction' (7, 8), with which he was admitted (habilitiert) as a Privatdozent of mathematics.

In this paper the ideas conceived in the earlier attempt on heat conduction, became mature and fertile. He studied the propagation of electromagnetic waves on a Riemann surface with two branches and obtained thus the first rigorous solution of a diffraction problem in the form of a complex integral, well suited for numerical calculation. W. Voigt, who had the chair for theoretical physics, became interested in the progress of this paper and used it later in his own way. Soon after its publication (7) H. Poincaré took up this 'méthode extrêmement ingenieuse' in *Acta Mathematica*. In fact it is to-day regarded as a classic of theoretical physics.

Sommerfeld devoted himself with enthusiasm to his lecturing which covered a wide field in mathematics, including the theory of probability and the partial differential equations of physics. In 1895–1896 Klein gave a series of lectures on the theory of the spinning-top, which was the starting point of the well-known book *Die Theorie des Kreisels* by Klein and Sommerfeld. It is the most valuable and mature fruit of the collaboration of these two men and reflects the change in attitude to mechanics in general which Sommerfeld experienced during the long period of writing the four volumes. The first two volumes stress the mathematical point of view while the third and the fourth (finished in 1910, when Sommerfeld was already in Munich) deal with the applications to geophysics, astronomy and technology, using often simple intuitive arguments. This tendency was also due to Klein who wished to fertilize German technology by presenting mathematical methods in a plain and easy language; Sommerfeld became one of its foremost protagonists. His whole scientific development was in the direction from pure towards applied mathematics and empirical science. There was a moment when he had the opportunity to do experimental work. Voigt offered him an assistantship in his laboratory. But Sommerfeld declined, though with regret. Later he became convinced that his decision was right, and that he had made the best use of his gifts by concentrating on the theoretical interpretation and inspiration of experiments.

Sommerfeld's first encounter with a problem of actual experimental physics happened during this period when he became interested in the propagation of electromagnetic waves along wires (13, 14), as used by Hertz in his famous experiments. This problem was at that time of central importance for experimental physics, in a similar way as hollow guides are to-day. It had already been treated by Hertz for an infinitely thin wire, and Poincaré, J. J. Thomson, Rayleigh and Drude had tried to improve the solution by taking the finite diameter into account, yet without complete success. Sommerfeld (14) solved the problem rigorously and developed formulae which represent the dependence of the field on the properties of the material of the wire.

In 1897 Sommerfeld became Professor of Mathematics at the Mining Academy in Clausthal, in the Harz Mountains. There he had mainly to lecture on elementary mathematics. During this time he published a paper on the diffraction of X-rays where he applied his method of many-valued solutions to the case of an aperiodic pulse (17, 18). In Clausthal he became the editor of Volume V (physics) of the *Mathematical Encyclopaedia*, founded and directed by Klein;

a big task which was to absorb much of his time and strength for a long period.

In 1900 he was offered and accepted the chair of technical mechanics at the Technological Academy (Technische Hochschule) in Aachen. Thus he was compelled to concentrate for a few years on technological problems.

At first his new colleagues and students regarded him with some distrust as a mathematician, but he soon had the satisfaction of being recognized as a useful member of the staff, not only in the lecture room, but also for the practice of engineering. He was elected a fellow of the technological society, was consulted as an expert, and published some papers on engineering problems, as, for instance, on the dynamical aspect of the strength of materials (19), on the oscillations of dynamos (24), on the action of railway brakes (21). The most important paper in this series is that on the hydrodynamical theory of lubrication (23), in which older work by Petroff and Osborne Reynolds was developed and compared with new observations. He seems to have particularly enjoyed this evidence of the power of mathematical physics applied to an object which was previously regarded as inaccessible to exact reasoning.

But these technical diversions could not suppress his deeper theoretical interests. He turned his attention to electrodynamics and investigated the resistance of coils for alternating current (28, 38). Then he proceeded to a more fundamental problem, the dynamics of the electron (30, 31, 35, 37). At that time Lorentz's theory of the electromagnetic field and of the electron was generally accepted. It was based on the assumption of an aether at rest, defining an absolute system of reference and capable of electric and magnetic excitation, described by Maxwell's equation for a field in empty space. The electrons were assumed to be rigid spheres carrying a fixed density distribution of electric charge. The problem was to find the equations of motion for such a rigid electron under the action of its own field and an external given field. J. J. Thomson had first shown that one obtained in this way an increase of inertia and suggested that possibly the whole mass of the electron might be of electromagnetic origin. This work had been improved by Abraham, Herglotz and others, and was now taken up by Sommerfeld in three voluminous papers (37), which displayed a great amount of mathematical skill to the solution of this problem. Using the method of Fourier transforms he found explicit formulae for the field of electrons in arbitrary motions and calculated the resulting force and the resulting moment, obtaining not only Abraham's formula for the mass of an electron in a quasi-stationary state of motion, but expressions valid for arbitrary accelerations, and he discussed even the case of velocities greater than that of light. But alas, just in the same year 1905, when the last of these three formidable papers appeared, it had become clear that no particle could ever move faster than light: it was the year of Einstein's first paper on relativity, which removed the fixed aether, the rigid electron and with it the foundations of Lorentz's theory.[3] A situation like this is a test not only of a man's power of scientific judgment but also of his character. It is not easy to abandon a line of research in which a tremendous

[3] H. Poincaré's earlier publication of 1904 was unknown in Germany.

amount of work has been invested, as can be seen from the attitude of the great Lorentz himself and of some of his followers, like Abraham. But Sommerfeld burned his boats and became a convinced relativist. It is, however, amusing to remark that just that part of Sommerfeld's paper which deals with electrons moving faster than light, and which seemed finally doomed by relativity has, much later, experienced a resurrection. For it can be applied, with some modification, to electrons penetrating material bodies where the velocity of light is so reduced that it is slower than the electrons. In 1934 R. A. Tcherenkov discovered this phenomenon which roughly corresponds to the conical shock-wave predicted by Sommerfeld. In the optical volume (V) of his Lectures he has given a very elegant outline of the theory of the Tcherenkov radiation.

In 1906 Sommerfeld was offered the chair of theoretical physics in Munich, previously held by Boltzmann. He accepted and took with him his assistant Peter Debye, the first of a long line of brilliant pupils and collaborators. Sommerfeld had several opportunities to exchange Munich for other places, but he refused all offers, even a call to Vienna (1916) and to Berlin (1927) as Planck's successor. In Munich he felt he was at the right place; here he could lecture on various branches of theoretical physics, and not only on well established subjects but also on still disputed problems. His seminars and colloquia attracted students and young scholars from far away and made Munich a centre of theoretical physics.

Sommerfeld's first contribution to relativity deals with an objection raised in a discussion by W. Wien (39). It has to do with the fact mentioned above in connexion with the Tcherenkov radiation: in dispersing media the phase velocity of light may be greater than *in vacuo*, in apparent contradiction to relativity. Sommerfeld showed that the velocity of a signal, which an observer with an infinitely sensitive detector would measure, is exactly equal to that *in vacuo*; this first signal then develops to a train of waves, the phases of which may travel faster than waves *in vacuo*, but cannot be used for signalling.

In this work he uses the method of complex integration with a virtuosity which is found in many of his later papers. Five years after the first publication of this result (1909) he gave a detailed account of it (65), followed by a still more elaborate paper by his pupil L. Brillouin (*Ann. Phys.* (4), **44**, 177, 203 (1914)).

In 1908 Sommerfeld attended the Congress of Science (Naturforscher-Versammlung) at Cologne and was deeply impressed by Minkowski's lecture in which he gave the first account of his four-dimensional representation of electrodynamics and relativistic mechanics. In the same year Sommerfeld had already started his first course of lectures on relativity, from which several publications emerged: a small one in 1909 (44) on the composition of velocities and two big ones in 1910 (47) on four-dimensional vector algebra and vector analysis. The latter papers have been so thoroughly assimilated by theoretical physicists that few will remember how much of the present technique and terminology of relativity is due to them; for instance, the expressions 'four-vector' and 'six-vector' which Sommerfeld suggested instead of Minkowski's